Assisting Defined-Benefit Pension Plans

John M. Mulvey
Department of Operations Research and Financial Engineering, Bendheim Center for Finance, Princeton University, Princeton, New Jersey 08544, mulvey@princeton.edu

Koray D. Simsak
Faculty of Management, Sabanci University, Orhangazi, 34956 Tuzla, Istanbul, Turkey, ksimsak@sabanciuniv.edu

Zhuojuan Zhang
BlackRock Financial Management Inc, New York, New York 10055, zhuojuan.zhang@blackrock.com

Frank J. Fabozzi
School of Management, Yale University, New Haven, Connecticut 06511, frank.fabozzi@yale.edu

William R. Pauling
Hartford Investment Management, Hartford, Connecticut 06105, bill.pauling@himco.com

The defined-benefit pension system poses substantial long-term risks for the U.S. economy. We describe a flexible asset-liability management (ALM) system for pension planning. The primary goals are to improve the performance and survivability of the pension trust. We first employ a stochastic program for enhancing investment strategies in light of company and other goals and pension risk constraints. The results are linked with a policy simulator for further analysis. We illustrate the concepts via two disparate real-world companies. The first is a slowly growing auto company, and the second a profitable pharmaceutical enterprise. We show that a stochastic program can help in the process of discovering sound policy rules. The ALM system has been employed extensively throughout the world by a large global actuarial firm.

Subject classifications: finance; corporate; portfolio; programming: stochastic; simulation: applications; decision analysis; multiple criteria, risk.

Area of review: OR Practice.

History:Received May 2004; revisions received August 2005, July 2006, February 2007, August 2007; accepted August 2007.

1. Introduction

As the U.S. population ages, the financial health of corporate and state and local government-sponsored pension trusts will have a substantial impact on millions of retired Americans, as well as workers who plan to retire over the next decades. However, a majority of pension trusts in the United States (and other countries) have lost the large surpluses that existed in the late 1990s. The Pension Benefit Guaranty Corporation (PBGC) estimated the total deficit of corporate defined-benefit trusts as of June 2003 to be roughly $400 billion (see Kandarian 2003, Nesbitt 2003). The sharp losses can be attributed to three primary causes—decline in equity markets, relating to the 2000–2003 economic downturn, the commensurate decrease in interest rates, and poor planning by plan sponsors.

In this paper, we focus on improving the planning process. In particular, we show that a systematic planning approach can pinpoint trade-offs and discover improving solutions among the goals of the diverse stakeholders. For selected situations, a carefully constructed model can enhance the company's ability to pay retirees, reduce net present value of future contributions, and assist in anticipating difficulties for the pension trust. To illustrate practicality, Olson (2005) describes usage of our ALM system in 1999 in preserving Kodak's pension surplus during the 2000-2003 contraction. The approach builds on methods employed by actuaries when they conduct pension valuation projects.

Large U.S. pension trusts fall into one of two traditional categories: (1) defined contribution (DC) plans, and (2) defined benefit (DB) plans. Under the former, the sponsoring entity (company or nonprofit agency) contributes a predetermined amount into the employee's tax deferred account. The individual takes responsibility for investment decisions within the context of the allowable options for the plan. We do not focus on DC plans in this paper (see Bodie et al. 1988). However, the modeling concepts can be applied to individuals who are planning for retirement via DC plans (see Berger and Mulvey 1998).

DB pension planning is complicated by the presence of several key uncertainties, a set of complex regulations, diverse stakeholders, and large pools of assets. In fact, in several mature industries, pension assets are many times
the size of the company's market value. For example, at the end of 2005 General Motors combined pension trusts had over $95 billion in assets, while GM's capitalization was roughly $11 billion. Many airlines occupy a similar situation. Accordingly, pension performance can have a significant impact on a company's long-term financial condition.

A pension trust can be interpreted as an inventory of assets (stock, bonds, etc.) set up to support an uncertain stream of future, long-term liabilities. The simple story goes as follows: the sponsoring company makes contributions and expects that the plan's assets will grow adequately to render future cash flows—that is, payments to the retirees. The planning process becomes complicated, however, because uncertainties arise at numerous junctures, the stakeholders have differing interests, and there can be conflicting loyalties among the decision makers. A company's workforce is a dynamic entity—workers are hired, fired, quit, move up the ranks, and die. The overall company size, its growth projections, and the age of the employees affect the pattern of future cash flows. Actuaries are trained to make estimates of these factors in light of company-specific data and extensive regulations for calculating cash outflows (liabilities). The present value of cash flows must be evaluated annually to fulfill a company's need to conduct a DB pension valuation. Future discount rates provide a significant uncertain element.

Contribution decisions by the sponsoring company are largely driven by actuarial rules and traditions that have been established to protect trust beneficiaries. Pension administrators must conduct an annual valuation by professional actuaries. Roughly speaking, the formula is: pension value = market value (assets) − present value of liabilities. Liabilities are the projected payments to the beneficiaries (retirees). The ratio of market value of assets to present value of liabilities is referred to as the funding ratio. A contribution is required whenever the plan falls below certain thresholds. According to the U.S. Employee Retirement Income Security Act (ERISA), which was instituted in 1974 and amended several times, if the market value of assets is less than 90% of the present value of liabilities, deficit reduction contributions (DFC) are required to achieve full funding in three to seven years. DFC is not required for underfunded plans that are at least 90% funded, however, they still need to make a contribution above the service cost by amortizing the unfunded liability over a period of five to 30 years. Furthermore, all underfunded trusts must contribute the present value of benefits accrued during the year (called service or normal cost).

We estimate liabilities over a carefully-specified set of probabilistic scenarios [4]. Scenarios are crafted so that pension assets and liabilities (and sponsoring company) can be simulated in a consistent fashion. For example, within each scenario, changes in interest rates drive both the returns of fixed-income assets and the calculation of present value of liabilities. The developed multifactor scenario generation process is described in Mulvey (1996) and Mulvey et al. (2000).

A second (option) set of uncertainties focuses on the sponsoring company's ability to pay future liabilities. Severe difficulties arise when the trust is largely underfunded and the company is simultaneously experiencing financial distress. A healthy company with a relatively small pension will not pose a large concern regarding paying future liabilities. Alternatively, a company whose business is closely linked to the economy, such as autos or airlines, will have difficulty rendering contributions during periods of economic contraction. Thus, we evaluate the company's ability to pay future liabilities. Similarly, the system considers corporate taxes. For example, company growth is projected on an after-tax basis. Alternatively, the planning system can be run in a direct fashion without reference to the sponsoring company—via a stand-alone pension ALM system. In the latter case, the board of directors will evaluate the intersecting interests of the company and pension trust outside the ALM system.

Previous research on pension planning considers asset decisions either on a relatively independent basis (Black 1989, Bodie 1990, Peskin 1997) or in conjunction with liability projections (e.g., Bogentoff et al. 2001, Dert 1995, Dempster et al. 2006, Boender et al. 1998, Kouwenberg and Zenios 2001, Mulvey et al. 2000, Ziemba and Mulvey 1998, and Zenios and Ziemba 2006) provide implemented examples. These models show the advantages of dynamic, adaptive investment strategies.

In many studies, contribution decisions appear as endogenous variables; a company will make a contribution only when required. Thus, contribution decisions are functions of the investment and liability variables and resolution of the uncertain asset returns. In selected cases, a pension administrator should evaluate voluntary contribution and possible borrowing decisions (Amott and Bernstein 1999, Bader 2003). It is generally agreed that pension contributions limit the ability of a company to make capital investments, thus inhibiting economic growth (Rauch 2006). In contrast, there are tax advantages to voluntary
contributions. Extending the model to include contribution and borrowing as decision variables builds on research involving enterprise risk management. Especially, the enterprise model will identify economic scenarios that lead to severe difficulties for the sponsoring company. Alternatively, the planning system can be implemented as a standalone entity; many companies have chosen this structure.

A specialized approach for pension funding aims to reduce or eliminate the variability of contributions through multiperiod immunization strategies.1 (See Dempster et al. 2006, Ryan 1997, Leibowitz et al. 1992.) Here, the sponsoring company purchases fixed-income instruments so that adequate funds are available to pay future liabilities for all possible scenarios. Four difficulties arise. First, payments to beneficiaries are generally indexed (directly or indirectly via salary increases) to future inflation. Second, future outflows may grow faster or slower than anticipated due to changing employee composition. Thus, it may be impossible to match all cash flows with available fixed-income instruments. In these instances, contribution decisions will depend on the resolution of future uncertainties because there is no easy way to completely hedge the future company-size risks (Black 1989, Ippolito 2002). Fourth, movement to bonds by a large segment of investors can cause prices to rise (and returns to decrease).2 The immunization strategy presents a constrained case of the described ALM system.

The health of a large corporate pension affects many people, with partially differing interests. These include retirees, workers, and senior management in the sponsoring company, shareholders, and the public.3 Due to the range of interests, our model considers a relatively large set of objective functions (§2). Pension administrators are often more receptive to one or more of the modeling goals, over the others. The ALM system identifies the rewards and risks to the various parties and seeks for nondominated solutions among pairs of objectives, subject to computational issues such as nonconvexity.

The trade-offs among the objectives become critical when the pension is severely underfunded, when the trust is large relative to the sponsoring company's resources (measured by market capitalization in our model), and when the company is growing slowly or shrinking. Unfortunately, some decision makers have dual and possibly conflicting loyalties, as company executives and as pension administrators. Some researchers suggest that, due to this and other potential conflicts, investment decisions should be made as conservatively as possible via immunization. However, the expected cost of immunization will be higher for the sponsoring company than alternative long-term strategies, such as taking on some equity exposure. Individuals may possess multiple stakes, e.g., long-term employees who own substantial company stock. The ALM system helps evaluate these issues.

A primary aim of the planning model is to avoid, when possible, the aforementioned worst-case situation.

Importantly, the ALM system draws on the implications of alternative investment strategies, such as a shift from a blend of equities and bonds to fixed-income assets (a.k.a. immunization).

The remainder of this paper is organized as follows. In §2, we describe a multiperiod stochastic approach for modeling a pension trust. We advocate a two-step approach. The first step involves building a multistage stochastic program. This model is a stylized depiction of the problem, without the clutter of many details, so that numerous stochastic optimization runs can be conducted. The results provide benchmarks for the detailed policy simulator—the second step. In §3, we present illustrative results for two U.S. companies. We have chosen firms with widely differing characteristics to depict a range of issues that arise when an ALM system is implemented. Next, we discuss the results and implications for alternative company strategies for managing their pension plan. We conclude in §4 with directions for future work in this important societal application.

2. The ALM Pension-Planning System

This section defines the mathematical framework for modeling a DB pension trust. The first phase involves setting up a multistage stochastic program. In the second phase, we construct a policy simulator, partially based on the results of the stochastic program. The simulator extends the system's range by providing features that are difficult to include in the stochastic program such as actuarial rules.

The developed model can be implemented at three levels of abstraction, depending on the needs of the pension administrators. First, the model can be employed for optimizing the asset allocation process for long-term investors, who are interested solely in the performance of the plan's assets (without regard for liabilities). Next, the model can optimize on asset performance within the context of the company's liabilities. The resulting ALM system evaluates the ability of the assets to keep up with the plan's future cash requirements, as well as future mandatory contribution decisions. In the third level, we integrate the company and the pension trust via the tools of enterprise risk management. This most comprehensive approach requires a relatively well-trained user, as well as a clear idea of resolving potential conflicts between the company and its pension trust. In this paper, we present the enterprise model, but the other two versions, which are special cases, have proven beneficial for many companies.

The enterprise system addresses the major decisions involving pension planning: (1) investment guidelines and policies, including asset allocation, which are made by the plan trustees typically in consultation with a pension consultant, (2) the sponsor's contribution policy, and (3) the corporate borrowing policy for deciding whether to borrow funds to make a plan contribution. Previous models have treated these decisions as relatively independent of each
other. The enterprise model extends the approach in Mulvey et al. (2003) by including company borrowing/contributions as formal modeling elements.

To start, we define the target planning horizon as $T = \{0, 1, \ldots, \tau, \tau + 1\}$. We focus on the pension trust’s position and the value of the company at the beginning of period $\tau + 1$. Investment and contribution decisions occur at the last instant of each time stage. Asset classes are defined by the set $A = \{1, 2, \ldots, l\}$, with asset class 1 representing cash. The remaining asset classes can include broad investment groupings such as stocks, long-term government or corporate bonds, and foreign equity. The asset classes should track well-defined market segments. Ideally, the co-movements between pairs of asset returns would be relatively low so that diversification can be done across the asset classes.

As with most implemented ALM models, uncertainty is depicted by a set of distinct realizations, called scenarios, $s \in S$. The scenarios may reveal identical values for the uncertain quantities up to a certain period, i.e., they share common information history up to that period by means of a scenario tree. There have been many successful efforts to generate representative scenarios for pension trusts (Hoyland and Wallace 2001, Kouwenberg and Zienios 2001, and Ziemba and Mulvey 1998). We employ the multistage CAPL:INK scenario generator, which has been in active use by Towers Perrin and its Tillingham business since 1993. Details of this generator appear in Mulvey (1996) and Mulvey et al. (2000).

We assume that the investment portfolio is rebalanced at the end of each period. For convenience, dividends and interest payments are reinvested in the originating asset class.

For each $i \in A$, $t \in T$, and $s \in S$, we define the following parameters and decision variables:

**Parameters**

- $r_{i,t,s} = 1 + \rho_{i,t,s}$, where $\rho_{i,t,s}$ is the rate of return for asset class $i$, in period $t$, under scenario $s$.
- $b_{i,s} = 1 + \gamma_{i,s}$, where $\gamma_{i,s}$ is the percent growth rate of the company in period $t$, under scenario $s$.
- $h_{i,s}$ Payments to beneficiaries in period $t$, under scenario $s$.
- $\pi_s$ Probability that scenario $s$ occurs — $\sum_{s \in S} \pi_s = 1$.
- $\chi_{i,t,0}^s$ Amount allocated to asset class $i$, at the end of period 0, under scenario $s$, before first rebalancing.
- $\chi_{i,t}^s$ Value of the company at the end of time period 0.
- $\psi_{i,s}^*$ Borrows costs for period $t$, under scenario $s$.
- $\sigma_{i,t}$ Transaction costs for rebalancing asset $i$, period $t$ (symmetric transaction costs are assumed).

**Decision Variables**

- $x_{i,t,s}$ Amount allocated to asset class $i$, at the beginning of period $t$, under scenario $s$, after rebalancing.
- $x_{i,t,s}^+$ Amount allocated to asset class $i$, at the end of period $t$, under scenario $s$, before rebalancing.
- $x_{i,t,s}^{BUY}$ Amount of asset class $i$ purchased for rebalancing in period $t$, under scenario $s$.
- $x_{i,t,s}^{SELL}$ Amount of asset class $i$ sold for rebalancing in period $t$, under scenario $s$.
- $\gamma_{i,s}$ Total amount of assets in pension plan at the beginning of time period $t$, under scenario $s$.
- $\gamma_{i,s}^*$ Value of the company after a contribution is made in period $t - 1$, under scenario $s$.
- $\gamma_{i,s}^{CONTR}$ Value of the company at end of period $t$, before contribution is made in period $t$, under scenario $s$.
- $\chi_{i,t,s}^{CONTR}$ Amount of cash contributions made at end of period $t$, under scenario $s$.
- $\chi_{i,t,s}^{BORR}$ Amount of borrowing by the company at end of period $t$ for use in pension plan, under scenario $s$.

### 2.1. Multistage Stochastic Program

We present the deterministic equivalent of the stochastic program in terms of the previous variables.

**Model [MSP]**

Maximize $U(Z_1, Z_2, \ldots, Z_t)$.

subject to

- $\sum_{i \in A} x_{i,t,s} = x_{i,t,s}^+ \quad \forall s \in S, t = 1, \ldots, \tau + 1$.
- $x_{i,t,s}^+ = r_{i,t,s,x_{i,t,s}} \quad \forall s \in S, t = 1, \ldots, \tau, i \in A$.
- $\gamma_{i,s}^{CONTR} = \gamma_{i,s}^{CONTR} - e_{i,s} - c_{i,s}(x_{i,t,s}^{BORR})$.
- $\sum_{i \in A} x_{i,t,s} = x_{i,t,s}^+ + x_{i,t,s}^{BUY} (1 - \sigma_{i,t,s}) - x_{i,t,s}^{SELL}$.
- $\sum_{i \in A} x_{i,t,s}^+ = x_{i,t,s}^+ + \chi_{i,t,s}^{CONTR} (1 - \sigma_{i,t,s}) - x_{i,t,s}^{BORR}$.
- $\gamma_{i,s}^* = \gamma_{i,s}^{CONTR} = \gamma_{i,s}^{CONTR} + x_{i,t,s}^{BORR}.$
- $\gamma_{i,s}^* = \gamma_{i,s}^{CONTR} = \gamma_{i,s}^{CONTR} + x_{i,t,s}^{BORR}.$
- Risk $\{Z_1, Z_2, \ldots, Z_t\} \leq \text{Risk}_{\text{max}}$.

A generalized network graph of the model appears in Figure 1. This graph depicts, for a given scenario, cash...
flows across time for each asset and the company. Although not all constraints can be incorporated, the graphical form allows managers to readily comprehend the model’s structure.

Constraint (2) represents the total value of assets in the pension trust at the beginning of period $t$. Constraint (3) depicts wealth accumulated in asset $i$ at the end of period $t$ before rebalancing. The growth of the company is shown in Equation (4). Note that the growth parameters derived via CAPM will deviate from their true values. This is because decisions such as contributions and borrowing change a company’s leverage, affecting its correlation with the market. Franzoni and Martin (2006) show that companies with severely underfunded pensions have lower stock returns than firms with healthier pension plans for at least five years after the first emergence of the underfunding. Our simplification can be easily extended by calculating growth as a function of certain decisions. Equation (5) defines the balance constraint for the company by subtracting the cash contributions at each period. From a practical perspective, the company value, which is measured by market capitalization in our model, may be seen as the value of investable assets in the corporation. This would be a natural interpretation of the full negative impact of contributions and debt payments on this value, as in constraint (5). However, as explained above, the growth rate is modeled exogenously in our framework and should potentially be lower during periods of high pension plan contributions, which is also supported by Franzoni and Martin (2006). Therefore, we adjust for this upward bias in $y^*$ variables through a full negative impact of contributions on the company value ($y$ variables). Alternatively, one could model the value of investable assets of the company because this value would naturally be fully reduced by contributions. However, this choice would require an endogenous modeling of return on investments (ROI) for the company.

The flow-balance constraints for all asset classes except cash are given by (6). This constraint guarantees that the amount invested in period $t$ equals the net worth for that asset. Constraint (7) represents flow balance for cash; the benefit payments and cash contributions are accounted for in this constraint. Nonanticipativity constraints are given in (8), ensuring that the scenarios with the same past will have identical decisions up to that period (Kall and Wallace 1995). Although these constraints are numerous, solution algorithms can take advantage of their structure.

The risk-based constraints appear in (9). For consistency, we require a minimum funding ratio for the pension at the end of the planning horizon. This constraint ensures that the trust is sufficiently healthy at the end of the planning horizon. Further risk-based constraints are discussed later.

For regulatory purposes, a pension trust must conduct annual evaluations to help assess the plan’s ability to pay its beneficiaries in the future and to evaluate required contributions. To this end, actuaries calculate the surplus or deficit as follows:

$$Sw_{i,t} = x_{i,t}^{T} - \text{present value } (b_{i+1,t}, b_{i+2,t}, \ldots),$$

where the present value calculation depends on regulatory requirements. There are two types of calculations—accumulated benefit obligations (ABO) and the projected benefit obligations (PBO). Briefly, the ABO measures current obligations, whereas PBO includes anticipated future
Factors such as salary changes. For actuarial calculations that drive contribution requirements, a smoothed value of assets is used instead of the market value of assets; however, herein, we rely on the market valuation to reduce the impact of smoothing of asset returns in the process. Van Binsbergen and Brandt (2006) show the negative impacts of smoothing on the liability valuation. In the model, we calculate ABO at each node in the tree, i.e., the liability valuation is updated as time passes taking into account the economic factors and the growth of the company; hence, it includes random variables involving future benefit obligations. ABO valuation is chosen over PBO valuation because the former is the required liability valuation for funding calculations defined by ERISA. Bodie (1990) and Bader (2003) also argue that PBO would be inappropriate because it accounts for future salary increases which should not be a corporate liability that the employer has guaranteed. For the stochastic program, we do not add hard constraints to force a contribution to be made. The optimization model will render a contribution when it enhances the value of the objectives. Because we intend the stochastic program to capture the economics of the whole process, we refrain from imposing the ERISA rules on contributions at this phase. The policy simulator will evaluate alternative contribution rules that can be imposed by the regulator.

### 2.2. Corporation and Pension Goals

The overall goal of the stochastic program is to determine a suitable compromise strategy for rendering the major decisions involving the pension trust. At the enterprise level, the approach seeks to improve the shareholder value for the sponsoring company, while putting the pension trust on a sound basis. Setting priorities for the integrated corporate pension trust presents a complex, potentially controversial issue. Generally, in practice, pension administrators define an efficient frontier between risk and expected surplus at the end of the planning horizon. There are several alternative definitions for risks—including variance, downside risk, value at risk, and conditional value at risk. Corporate executives, on the other hand, face the issue that shareholders can conduct their own diversification, thereby reducing the need for risk aversion in corporate pensions. Risk aversion arises as endogenous variables, due to higher borrowing costs, possible bankruptcy issues when the firm encounters economic distress and issues relating to perception. Froot and Stein (1998) and Jarrow (2003) provide approaches that address these issues. Also, there are likely to be externalities for certain industries, such as banks and insurance companies, thus requiring additional risk constraints due to regulations. For pensions, the safety of a trust is dictated by long-standing regulations, such as ERISA and the prudent man rule.

The overall utility function (1) emphasizes the multi-objective optimization nature of the problem in the real world. The goal is to find an acceptable compromise among the competing objectives. For example, we could define an expected utility function of the combined wealth (company plus pension value) at the horizon (period \( t+1 \)), as defined in Mulvey et al. (2003). The model does not take a fixed position on the required degree of investment risks, but rather provides a flexible template for projecting the implications of current decisions on the future environment for the pension trust.

Next, we define a set of objective functions. These goals depict alternative views on the state of the pension trust to optimize its value for the company’s shareholders at the planning horizon. In practice, depending on the needs of the clients, some of the goals are selected for attention (only) over others and do not impact the model’s recommendation. Further discussion on priorities takes place in §3.

The first objective evaluates the company’s shareholder value at the horizon and is formulated as:

\[
Z_5 = \sum_{t=1}^{T} \pi_t^s \mathbb{E}_{t}^{s-1} r_t
\]

where \( \mathbb{E}_{t}^{s-1} r_t \rightarrow \mathbf{E}_{t}^{s-1} \) is the discounted cash flow at time \( t \), with \( r_t \) the joint distribution function. As in insurance, maximizing shareholder value is equivalent to maximizing risk-adjusted growth of earnings. The classical goal—maximizing expected company value (separate from the pension)—is often proposed by corporate planners. We may employ this function in conjunction with a series of constraints to protect the pension trust and, by implication, safeguard the company’s promises. For simplicity, we treated the company in a straightforward fashion, without significant corporate decisions such as leverage or M&A. This aspect can be expanded in future versions.

The planning system must address the needs of diverse stakeholders—current and future retirees, stockholders, the PBGC, and the public. As discussed, the general public will become liable if the PBGC is unable to pay its liabilities due to a severe increase in bank ruptcies. As a second risk measure, therefore, we focus on the likelihood of a large payment, called excess contribution, to the pension trust at any point during the planning period:

\[
Z_2 = \mathbb{P}_{t}^s \left( \mathbb{E}_{t}^{s} - \alpha \cdot Y_{t, n}^s \right) \cdot \mathbb{I}_{t}^s
\]

where \( \alpha \) equals a substantial fraction of the company’s total capital at the time of the contribution (Bogeatoft et al. 2001). This function approximates the company’s ability to service its pension trust under severe circumstances.

A related measure, aimed at the distribution tails, is the probability of a bankruptcy: \( Z_3 = \mathbb{P}_{t}^s \left( \mathbb{E}_{t}^{s} \geq Y_{t, n}^s \right) \) for any time \( t \) and scenario \( s \). Here, we assume that the company is unable to make a contribution larger than the company’s current market capital, and must turn to a bankruptcy court for relief. This stylized study makes this assumption recognizing that bankruptcy is dealt with differently in practice. It is likely that the bankruptcy decision will be made well before the mandatory contribution exceeds the company’s market capitalization.

The next function measures the volatility of future contributions for the corporation:

\[
Z_4 = \text{Std} \left( \mathbb{E}_{t}^{s} \mathbb{I}_{t}^s \right), \quad \forall n \in N
\]

where \( \mathbb{I}_{t}^s \) is the scenario tree (§3). In most cases, this function depicts a secondary issue to consider, once
the other goal values have been settled on. Lower volatility of contributions is desirable because there will be reduced impacts on the company's profits and the shareholder value. This function is especially meaningful for situations where $Z_1$ and $Z_2$ cannot be defined because the value of the company is several times larger than that of the pension plan.

The fifth function is the risk-adjusted discounted value of future contributions (Black 1995). This objective provides a measure for the long-run cost of the pension trust: $Z_5 = \sum_{s \in S} \pi_s \sum_{t \in T} X_{t,s} \left(1 + r_t\right)^{-t}$, where the risk-adjusted discount rate equals $r_t$, and is based on actuarial judgment.

Another objective, aimed at the pension trust, is to maximize (expected) surplus wealth at the end of the planning horizon: $Z_6 = \sum_{s \in S} \pi_s S_{w,s}$. This function focuses on the investment strategy and contribution policy of the pension trust so that the highest average surplus value is achieved.

Lastly, we compute the probability that the company will make an excess contribution, defined as in the second objective. This function is calculated as follows: $Z_7 = \sum_{s \in S} \pi_s \pi_s$, where the indicator is $I_s = 1$ if there exists an excess contribution under scenario $s$, and zero otherwise.

Due to the complexity of pension trust planning, we pose the seven aforementioned objectives. Likely, various stakeholders will be interested in other metrics; these can be added as variants of the ALM system. We have found that there are advantages to depicting the problem as widely as possible so that the various stakeholders will be able to understand the real-world trade-offs. Section 3 discusses two implementations, suggesting a ranking of the functions in conjunction with our two-stage process. As mentioned, the pension system has been implemented in the context of the pension asset and liabilities without reference to the sponsoring company in the model.

2.3. Policy Simulator

We employ a stochastic policy simulator for the second stage of the planning process. After solving the previous model, the simulator conducts further analysis and optimization runs. Motivating this step is the difficulty in implementing the recommendations of a complex stochastic program, including related statistical estimates, and performing sensitivity analyses. We have found that a policy simulator greatly improves the practicality of the planning system.

Policy simulators are relatively easy to implement, especially via software such as @Risk or Crystal Ball, and can include details that are difficult to portray in a stochastic program. For example, actuarial requirements involve calculating surplus wealth of a pension trust via a complex set of rules, with several smoothing factors and discrete conditional tests. Actuarial rules do not lend themselves to convex functions. To overcome these barriers, a policy simulator can provide important capabilities.

For a stochastic simulator, however, a primary challenge is to discover efficient and robust policy rules. An early example of a significant policy rule is the fixed-mix investment strategy, whereby a long-term investor is best served by rebalancing his portfolio to predetermined proportions at each time period. The fixed-mix investment rule can be shown to be an optimal process for long-term investors under a series of restrictive assumptions. Formally, a policy rule is a nonanticipatory process that can be implemented in the simulation without need for the scenario identifiers. Policy rules guarantee that scenarios with the same past will have identical decisions up to that period as long as the rule itself depends only on current and previous state variables. Decisions at time $t$ are a function of the state of the system at time $t$. Of course, it is natural to add accounting state variables that track the historical time path of the system up to time $t$; these variables can play significant roles.

The following formula shows the relationship of the policy rules to the investment variables in the model: $x_{t+1} = f_t(\psi_{t+1}, \psi_{t+1}, \ldots, \psi_{t+1})$, where the state of the system at time $t$ is indicated by the state variables, $\psi_{t+1}$, $t = 1, 2, \ldots, L$. For example, a policy rule might focus on the plan's surplus wealth. Here, we might define $\psi_{t+1} = SW_t$. As an example policy, we could target a fixed percentage of the surplus funds into equity assets, with the remainder invested in bonds. This rule is known as surplus-equity (see Peskin 1997). Another state variable is defined as the ratio of the market capitalization of the company to the market value of pension assets, or $\psi_{t+1} = y_t^e / y_{t+1}^b$. An example implementation of the policy based on this state variable is explained in Malvey et al. (2005).

Another policy issue involves setting contributions at each time period. Here again, the policy simulator requires a nonanticipatory formula. We found that a sensible policy is to require contributions to be a function of two derived variables, $x_{t+1}^{\text{CONT}} = f_t(\beta_{t+1}, SW_{t+1})$, where $\beta_{t+1}$ is the time remaining until the horizon date, equal to $(t + 1) - t$, and $SW_{t+1}$ is the surplus wealth (see §3.3).

3. Implementation

This section describes the implementation of the planning system for two large U.S. pension trusts. The sponsoring companies reside in industries with differing characteristics. As a consequence, the model is tailored to the company's unique circumstances. The first company operates as a large U.S. industrial firm in the Midwest, with an older workforce, a large number of retirees, and an income stream that is relatively sensitive to the general business environment. Unfortunately, the period 2000–2003 has not been kind to Company 1 in terms of its pension trust; its funding ratio has dropped from roughly 105% to about 70%. Yet, pension assets equal roughly half of the company's market value. The company’s average annual after-tax growth rate equals 7% and its correlation with S&P 500 is 89%.

The second company invents, manufactures, and distributes drugs and related health-care products in the global
marketplace, with headquarters in New Jersey. Given trends in demographics and the corresponding growth of the health sector, the profit of this company has less dependence on the general economic environment than Company 1. We model the relationship between the S&P 500 return and Company 2's return with correlation 30%. From the corporate planning staff, we estimate the geometric growth of after-tax earnings at 10% per annum. Its pension, whose assets amount to one-fifth of the company's market value, is relatively healthy (90% funding).

In each case, we evaluate various contribution and investment strategies. The primary objective is to maintain or increase the funding ratio to 90%, while paying retirees and minimizing the impact of pension contributions on the company's future earnings. The 90% target seems a realistic goal given the current degree of underfunding of most of the pension plans we are investigating. Moreover, current ERISA rules require that deficit reduction contributions (DRC) are enforced for plans with ABO-based funding levels less than 90%. Therefore, it should be viewed as a state of "health" that we are trying to raise the pension system to—the model finds the optimal strategy that should be followed to achieve such a status. From a technical standpoint, this can be perceived as an implementation of the end-effects methodology which provides an equilibrium state at the end of the modeling horizon (Birge and Louveaux 1997). Further sensitivity analysis on this target can be beneficial.

For this illustrative study, a nine-year planning horizon is chosen so that the company will have adequate time to preserve or improve its funding ratio. A critical issue is the ability of the company to fulfill its promise to pay beneficiaries over long periods. Under ERISA rules, DRC need to be paid in three to seven years. We extend this to nine years to provide flexibility for the extreme cases and to acknowledge the long-term nature of pension problems. For the stochastic process, we divide the planning period into three three-year intervals, with a fourth decision stage at the beginning of year 10. At that point, if needed, the sponsoring company is required to make a contribution to match the 90% funding ratio.

The projected payments to the beneficiaries are shown in Figure 2 over the next 25 years. These payments have been estimated by the firms' actuaries based on the current population of workers and retirees, dependent on the nature of the company and its respective demographic information. Actuaries have refined statistical approaches for valuing pensions, especially since the passage of ERISA.

At each node in the scenario tree, we calculate the projected changes in the cash flows, the accompanying pension surplus/deficit, and the corresponding funding ratio as required by pension regulations. The parameters in Equation (10) are based on the historical path and the corresponding cash flows (including paying current retirees, returns on the assets, contributions, etc.). We employed actuarial estimates of the changes in liabilities for each node in the scenario tree. For example, current inflation has an impact in future cash flows. To find the present value of liabilities, we employ the projected long government bond rates as the current discount factors.

Asset returns must be projected over the planning horizon. As mentioned, the CAPLink system generates future stochastic asset returns. The scenario generator consists of a set of stochastic differential equations for the principal economic variables, including spot interest rates, price inflation, equity returns, currencies, etc. For illustrative purposes, we select four major asset classes: short-term U.S. t-bills (cash), large U.S. equities (S&P 500), international equities (MSCI EAFE), and U.S. Treasury bonds (U.S. T-bonds). In this order, the initial asset allocation is 10%-30%-20%-40%. We also impose bounds on investments in these asset classes. Again in the same order, the lower bounds are 0%-25%-5%-15%, whereas the upper bounds are 10%-60%-20%-50%. In addition, the planning model allows the company to borrow funds for the pension trust as needed for use in period 1. Per company policies, the maximum borrowing cannot exceed 20% (5%) of the market value of Company 1 (Company 2). The single-period after-tax borrowing rate is assumed to be (cash return + 3%).

Interest payments will be made every three years, while the principal is paid at the beginning of year 10. Finally, we put an upper bound of 130% on the expected final funding ratio specifically for the risk minimization. Some of the risk objectives focus only on the excess contributions and, therefore, may lead to unnecessary contributions below the threshold. To avoid this, we restrict the contributions indirectly through this constraint, which also is sensible from a practical perspective because highly overfunded pensions will be unable to benefit from tax protections. As explained in §1, contributions are tax deductible until the plan is fully funded with respect to ABO. Beyond this level, not only this incentive disappears, but also the contributions are subject to a 10% excise tax. There are several proposals regarding the planned pension reform which recommend that the maximum tax-deductible contribution threshold should be relaxed to allow for sponsors to make voluntary contributions when the market conditions

![Figure 2. Anticipated payments to beneficiaries.](image-url)
are favorable (Kausch 2005). Not having a margin above
the full funding ratio may have adverse affects when inter-
est rates decrease sharply (leading to a higher valuation
of the liabilities). Parks (2003) claims that a 130% funding
ratio threshold for the tax-deductible contributions might
have helped in all but two periods in the last 100 years the
depression years (dramatic decreases in stock prices) and
the 2000–2003 period (dramatic decreases in stock prices
and decreases in interest rates). A more liberal ratio would
make borrowing for contribution more attractive, whereas
a more conservative ratio would increase the importance of
optimizing investment strategies. Kausch (2005) also points
out that the current administration’s funding reform pro-
posal can be interpreted as a maximum tax-deductible tar-
get funding ratio of 130%. For the minimum target
funding ratio, a sensitivity analysis on this upper bound
may provide further insights into the trade-off among opti-
nal funding, borrowing, and investment strategies. All cal-
culations are performed on an after-tax basis for the core
company. Remember that the pension trust does not pay
taxes, as long as it operates in accordance with government
regulations.

3.1. Company 1: Stochastic Programming Results
To handle the previous seven goals, we follow a multi-
objective strategy. We pick the two most important goals for
an efficient frontier analysis—one will represent a reward,
while the other will be a risk measure. For Company 1, we
pick $Z_1$ and $Z_2$ to be reward and risk objectives, re-
spectively. For each optimization, we record the values for the
remaining goals. For the set of 1,000 scenarios, the model
consists of roughly 68,000 variables and 78,000 constraints;
each point on the frontier takes four to seven minutes with
the quadratic version of CPLEX solver on a powerful PC.
The risk minimization is done in 18 quadratic program-
ming barrier iterations, whereas the reward maximization
is done in 16,317 dual simplex iterations. For this study, we
were able to solve the nonlinear program on a powerful PC
without difficulty, so that the model can be employed by
actuaries and asset consultants in practical settings. Due to
space limitations, we leave out the details, which are avail-
able from the authors. We present the efficient frontier in
Figure 3. A summary discussion follows.

First, the minimum-risk solution is much different from
the maximum-wealth solution, not only in terms of objec-
tive values, but also in terms of contribution strategies. The
former contributes large amounts early on, while the latter
renders most contribution in the last period. There is a large
range of possible outcomes over the seven goals. As might
be expected, the compromise solutions (middle of the fron-
tier) perform better with respect to the other objectives.

Borrowing is engaged for all runs. This result occurs due
to several causes, including tax benefits for asset growth
in the pension plan. For the current model, we allow bor-
rowing only at the beginning of period 1, to be paid back
at the end of the third period—nine years later. Although
liabilities grow at a 6% annual rate on average (compared
to 7% of the company value), the company has difficulty
keeping up with them. So, in addition to all contributions,
it chooses to install a relatively large amount of borrowing
for the pension trust at the first period and take advantage
of the tax benefits.2

The bankruptcy probability varies between 5% to 8%.
This probability appears slightly higher than the number of
bankruptcies that occurred in the older U.S. manufacturing
industries over the past decade.

As evident, Company 1 faces choices with a wide range
of outcomes. The second part of the planning process aims
to help in this context: $X_3$ takes up this issue.

3.2. Company 2: Stochastic Programming Results
Due to company’s lower dependence on the business cycle
(at least compared with Company 1) and its healthier fund-
ing ratio, the model recommendations are different than for
Company 1. Thus, making contributions early on is not as
desirable; see the intermediate solution.

Second, there are no scenarios resulting in a bankruptcy
or an excess contribution (greater than 30% of company
value). To address risks, we turn to $Z_1$—variability of
contributions. This goal helps smooth future earnings.
Figure 4 portrays an efficient frontier between goals 1 and 4.
Table 1. Values of the five goals for Company 2 for selected points on the efficient frontier.

<table>
<thead>
<tr>
<th>Point</th>
<th>Expected final company value</th>
<th>Variability of contributions</th>
<th>Expected final plan surplus</th>
<th>Downside risk on plan's final funding ratio (%)</th>
<th>Present value of contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>272,932</td>
<td>1,601</td>
<td>14,893</td>
<td>0.216</td>
<td>20,681</td>
</tr>
<tr>
<td>7</td>
<td>286,753</td>
<td>2,322</td>
<td>9,074</td>
<td>0.187</td>
<td>13,804</td>
</tr>
<tr>
<td>16</td>
<td>300,070</td>
<td>9,576</td>
<td>-1,856</td>
<td>0.057</td>
<td>8,016</td>
</tr>
</tbody>
</table>

See Tables 1 and 2 for further details. After observing the trade-offs, we propose that a reasonable compromise solution lies between points 6 and 10. Here, the expected company value is close to the maximum, around $295 to $300 billion, with a healthy pension surplus and a reasonable variability of future contributions.

3.3. Policy Simulation Results

In this section, we describe the second step in the planning process—discovering policy rules that are understandable to company executives and regulators, that adjust to the changing environment, and that achieve results that are reasonably close to the target goals identified by the stochastic program. An advantage of a policy simulator is that it can handle many stages and scenarios. Here, we simulate annual decisions through the nine-year horizon, instead of every three years as in the stochastic program. We also utilize an expanded set of 5,000 scenarios.

First, we define several contribution rules for Company 1. As before, we strive to achieve the 90% funding ratio at the planning horizon. So, a contribution pattern can be outlined by defining intermediate target ratios for each decision point. These targets can be designed such that they encourage/discourage earlier contributions.

Turning to the investment strategy, we study the results of the stochastic program. Here, the equity allocation is a function of the plan's funding ratio, at any given time. Thus, we divide the funding-ratio domain into three regions, each with its own investment perspective: (1) underfunded plans (less than 90%) should be more aggressive and therefore invest heavily in equity, (2) plans that are close to full funding (between 90% and 120%) should follow a more traditional strategy, and (3) plans that are overfunded (greater than 120%) should invest more conservatively to protect their surplus. These regions also determine allocations within the equities and the fixed-income instruments.

All first-period decisions, such as initial asset allocation, borrowing, and beginning contribution, are nonanticipatory and taken directly from the stochastic program that we are trying to imitate. In this case, we pick point 10 in Figure 3. The borrowed amount equals 20% of company value; the initial contribution amounts to 48% of the gap between the initial funding ratio and the final target funding ratio; the plan assets are invested 60% in U.S. equity, 30% in international equity, and 20% in U.S. T-bonds. Detailed results are left out due to space issues (available from the authors). Nevertheless, we show in Figure 3 how some solutions have fared with respect to the efficient frontier and present a summary discussion.

As a first observation, the SP results are moderately better than those of the policy simulator because policy rules are unable to take full advantage of the scenario structure. The stochastic program makes decisions, especially in the later years, that have a sizable impact on the objective function values. As the tree becomes larger, however, the stochastic program possesses greater uncertainties in the later years and therefore performs less well. Regardless, the results show, for an underfunded pension trust, the importance of making contributions based on the level of deficit and the time remaining to achieve a target funding ratio (90% in our tests). All policy variants for contribution produce similar results.

Next, we turn to Company 2 to conduct a policy analysis. Because Company 2 starts with a reasonable funding ratio, we follow a simple contribution strategy by treating the initial funding ratio as a threshold. Whenever the ratio falls below the threshold, the company brings it back to 90%
Table 3. The objective function values for Company 2 employing policy simulation.

<table>
<thead>
<tr>
<th>Objective functions</th>
<th>Point A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected final company value (billion $)</td>
<td>285.405</td>
</tr>
<tr>
<td>Variability of contributions (billion $)</td>
<td>2.761</td>
</tr>
<tr>
<td>Expected final plan surplus (billion $)</td>
<td>-2.829</td>
</tr>
<tr>
<td>Downside risk on plan’s final funding ratio (%)</td>
<td>0.083</td>
</tr>
<tr>
<td>Probability of a large contribution</td>
<td>0.009</td>
</tr>
<tr>
<td>Present value of contributions (billion $)</td>
<td>6.727</td>
</tr>
<tr>
<td>Probability of bankruptcy</td>
<td>0.002</td>
</tr>
</tbody>
</table>

by making a contribution. We follow the same investment strategy as Company 1. The initial decisions are in line with the stochastic program: the borrowed amount is equal to 5% of company value; there is no initial contribution; the plan assets are invested 60% in U.S. equity, 20% in international equity, and 20% in U.S. T-bonds. In this setup, we produce a sample solution in Table 3 and label it as point A in Figure 4. We achieve results close to the previous recommendations both in terms of the efficient frontier and the other goals. As such, we have devised “reasonable” policy rules.

There are several lessons to be gained from these implementations. First, the stochastic program plays an important role in the planning process: (1) by setting out specific targets for the policy simulator, (2) by illustrating a set of decisions that lead to improving the company, without overly tight constraints on the process, and (3) by analyzing the pension while approximating the sponsor. Because DB pension plans have become so large over the past decade for selected industries, there is need for closer coordination of the pension trust and the sponsoring company (formally or informally).

In addition, the empirical analyses show the following. First, the planning model provides a systematic approach for evaluating pension decisions on the sponsoring company. It can shape discussions regarding borrowing to improve the funding ratio of the pension trust. Second, at least for selected growing companies, borrowing may be a reasonable option and should increase shareholder wealth, although risks may increase. For severely underfunded companies, borrowing may be the only option. In these cases, regulators should monitor the plan so that the increased probability of a default does not cause further difficulties for the PBGC. Third, determining if a pension trust should make a contribution should depend not only on current valuation formulae, but also on chances that the pension trust will be able to operate successfully. The ALM system can help estimate this uncertainty. Also, the development of sound policy rules depends on the firm’s environment. For Company 2, the proposed policy rules gave the desired results—close to the stochastic programming results. On the other hand, the results of Company 1 were less satisfactory, farther from the efficient frontier, as compared to Company 2. In most cases, these could be detected in the stochastic program, where recommendations may have no significant pattern as a function of the state variables. As a slow-growing enterprise, Company 1 may adopt policies that depend not only on the funding ratio, but also on the plan’s size relative to the company. An example of such a policy rule for the telecommunication industry appears in Mulvey et al. (2005).

4. Conclusions and Future Directions

This paper describes a long-standing successful implementation of asset and liability management systems for DB pension plans. There are several contributions: (1) illustrating the usefulness of a widely applied anticipatory planning system for DB pension trust; (2) combining a stochastic program and a policy simulator for pensions; and (3) extending the ALM system via topics in enterprise risk management. The planning system provides a systematic approach for evaluating long-term pension performance/risk. For the model, the primary goals are to improve shareholder value (for example, by minimizing the present value of contributions), while enforcing constraints to protect the pension trust, pay beneficiaries over the planning period, and minimize risks that the trust will collapse due to insufficient funds. We advocate multiple risk measures due to the problem’s complexity: probability of making a large contribution, likelihood of a bankruptcy over the planning period, and related worst-case events. Rather than looking solely at static indicators, such as current funding surplus or deficit, we evaluate the pension trust (and possibly the company) over relatively longer time periods and across many dynamic scenarios. Temporal issues are evaluated, such as the impact of voluntary contributions (which will decrease current profit, but may increase future profits), or taking on greater investment risks for potentially long-term gains.

An important issue is to find the proper approach (formally or subjectively) for integrating pension decisions (asset allocation, contribution, and benefits) with the sponsoring company. These issues can be evaluated in a relatively independent fashion, possibly causing future problems when the company performs poorly during a period that the pension trust is running into a deficit such as the 2000–2003 downturn. The planning system aims to reduce the chance of these circumstances occurring simultaneously. Accordingly, the strategies should reduce risks that the company will be unable to meet its long-term obligations. Further, the costs of the pension trust may be lowered so that the company can grow more successfully. In the end, a healthy organization is more likely to maintain its pension trust in a sound manner than an unprofitable slowly growing company.

The ALM planning system can assist pension administrators and company executives in evaluating the pros/cons of strategic issues. For example, in a recent project with the U.S. Department of Labor, we employed the enterprise
ALM system to evaluate possible changes in regulations for DB pensions in the United States (Mulvey et al. 2005). As another example, there is much current interest in replacing DB plans with DC plans. The relative costs/benefits of alternative approaches to this conversion can be estimated. Regarding oversight issues, greater attention should be paid to "unhealthy" trusts by the stakeholders (regulators, beneficiaries, and shareholders). In this context, a relevant issue is to determine conditional policies for slightly underfunded pension trusts, such as those discussed in the telecommunication services industry. The planning system may help address related regulatory issues.

The enterprise ALM model can be extended in several directions. Decisions about the capital structure can be added, for example, to evaluate corporate borrowing decisions. A multifactor model would appear to be the next step. Also, asset categories can be readily added; typical implementations of the ALM studies for pension trusts often include 20 to 40 asset categories.

A complicated problem is to address the concerns of global companies with pensions in multiple countries. Balancing the needs of the individual trusts (with local rules and laws) with the corporate charter can be difficult. The enterprise version of the model might assist in this arena.

Last, the linkage of stochastic programs and policy simulation/optimization has applications in other OR domains. For example, supply chain planning often depends on complex policy rules. A stylized stochastic program may be able to assist in the search for improved rules. If so, applications of stochastic programs would be expanded because simulation models appear widely in industrial practice.

Endnotes
1. Motivating the move to reduce volatility is the concept that the company should avoid risks which are outside its special expertise. Thus, a company should no more invest retained earnings in a pension plan, than it should from an insurance company to reduce the costs of future insurable losses.
2. For example, prices of inflation-linked bonds have risen (and yields have dropped) in the United Kingdom since 2002 as pensions have shifted assets to fixed-income categories.
3. PBGC assumes control of a plan that cannot be funded, for example, by bankruptcy. Liabilities for highly paid employees may be reduced by the PBGC if funds are unavailable. Taxpayers may need to bail out PBGC under severe conditions. The organization is funded by fees from all DB U.S. plans.
4. The stochastic program helps set guidelines for policy simulation. Therefore, we choose to reduce problem size by dividing the nine-year planning horizon into three equal intervals. Carrión et al. (1994) adopts a similar strategy; however, because their model generates the final recommendation, they choose an uneven partitioning with shorter intervals in the more important earlier stages.
5. Cash flows change with the size of the workforce, realized inflation, and other events. We model these as a function of the nodes in the scenario tree.
6. For illustration, we assumed that the company pays a constant risk premium regardless of financial health of its pension and/or itself. One may model this parameter as a variable correlated with the growth rate of the company.
7. Although the corporation’s tax benefits from contribution or borrowing are not explicitly modeled in our framework, we limit growth of corporate assets by subtracting a factor for taxes, which provides a relative advantage for assets in the pension that can grow without tax liability. In practice, this advantage may not be present because corporate capital gains would only be taxed when realized (hence, a strategy of holding corporate equity and fixed-income assets in the pension would also avoid taxation). Tax incentive for borrowing to fund pensions has the strongest economic support for contributions that exceed the minimum required level. See Bodie (1950) for an extended discussion of this subject.

Acknowledgments
The first author was supported in part by the National Science Foundation through grant DMI-0623410. Without implication, the authors appreciate the assistance of the staff at Towers Perrin and the Policy Simulation Group, and the usage of Towers Perrin’s CAP-Link system for generating scenarios for the illustrative examples. William R. Pauling worked at Towers Perrin during the early computational research.

References


